

COSMOLOGICAL CONSTRAINTS ON THE SCALE OF SUPERSYMMETRY BREAKING

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ABSTRACT: We consider the cosmological and astrophysical constraints on the decay of massive $E_8 \times E'_8$ superstring axions associated with the hidden sector. We find that decay lifetimes greater than 1 s are ruled out by limits from nucleosynthesis, by limits on the distortion of the cosmic microwave and gamma ray backgrounds and by closure arguments. We conclude that $\Lambda > 1.2 \times 10^{13}$ GeV, where Λ is the scale of gaugino condensation in the hidden sector. This implies that the scale of supersymmetry breaking is greater than 10^{10} GeV. Significantly, our result agrees with the value of 5×10^{13} GeV for Λ obtained independently by setting supersymmetric scalar masses equal to m_W .

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$E_8 \times E'_8$ superstring theories are anomaly free and hence very attractive [1,2]. Witten has shown that in such theories there is a model independent $U(1)$ Peccei-Quinn symmetry. When this symmetry breaks one gets an axion with a decay constant related to the compactification scale because the non-renormalizable interactions of the model independent axion arise as a result of compactification [3]. Furthermore, there are several model dependent axion degrees of freedom.

In $E_8 \times E'_8$ models compactified on a Calabi-Yau manifold the model independent axion degree of freedom corresponds to $B_{\mu\nu}$, ($\mu, \nu = 1, \dots, 4$), the zero mode of the antisymmetric tensor field B_{MN} ($M, N = 1, \dots, 10$) which is essential for anomaly cancellation [3]. The model dependent axion is a linear combination of the zero modes B_{mn} ($m, n = 5, \dots, 10$) [4]. Neither of the corresponding PQ symmetries survive compactification and the above axions have decay constants related to the compactification scale [5].

If E'_8 breaks down to a non-abelian group there are two non-abelian groups today: $SU(3)_C$ and the non-abelian subgroup of E'_8 . The model independent and the model dependent axion degrees of freedom rearrange themselves to give two physical axions— the QCD axion a and the E'_8 axion a' , with decay constants close to the compactification scale [6]. The energy density of the axions is proportional to $\Lambda^\alpha F_a^\beta$ ($\alpha, \beta > 0$), where Λ is the scale at which the underlying group gets strong. The QCD axion with $F_a > 10^{12}$ GeV and $\Lambda \sim 200$ MeV will not have decayed by today and suffers an energy density problem. In this paper we study the cosmological and astrophysical constraints on the energy density of the E'_8 axion and on its decay to photons to give limits on Λ for the E'_8 axion. We find that Λ should be greater than 1.2×10^{13} GeV. Λ is also the scale of gaugino condensation in the hidden sector which breaks supersymmetry [7]. Since the masses of supersymmetric particles in the observed sector are approximately $10^{-1} \Lambda^3 / M_{Pl}^2$, this corresponds to supersymmetric particle masses greater than 1 GeV. The scale of supersymmetry breaking is given by $M_S \sim (\Lambda^3 / M_{Pl})^{1/2}$. Thus, M_S should be greater than 10^{10} GeV. Our result agrees with $\Lambda \simeq 5 \times 10^{13}$ GeV obtained by setting $m_{\text{gaugino}} \simeq m_W$ [7]. We find it significant that the result obtained by us using cosmological and astrophysical constraints agrees with the independent requirement from particle physics.

In a companion paper [8], we set Λ equal to 5×10^{13} GeV and study the dilution of the baryon asymmetry due to axion decays. We find that for this value of Λ , most models of baryogenesis can not tolerate the large dilution (by a factor of 10^7) of the baryon asymmetry.

World sheet instanton effects may add terms to the potential of the model dependent axion [9,10] for most Calabi-Yau spaces (though not all). In such a case, the QCD axion will not get its mass purely from an $F\tilde{F}$ term but can have large contributions from other

non-derivative couplings. The E'_8 axion will be almost equivalent to a_1 below [11]. The mass of the E'_8 axion will probably not be greatly affected. Hence, we ignore these effects below.

II

In an $E_8 \times E'_8$ superstring theory compactified on a Calabi-Yau manifold and in which the E'_8 breaks down to a non-abelian group, the axion lagrangian in 4 dimensions is given by [6]

$$L = \frac{1}{2}(\partial_\mu a_1)^2 + \frac{1}{2}(\partial_\mu a_2)^2 + \frac{1}{32\pi^2 F_1} a_1 (g^2 F \tilde{F} + g'^2 F' \tilde{F}') + \frac{1}{32\pi^2 F_2} a_2 (g^2 F \tilde{F} - g'^2 F' \tilde{F}') \quad (1)$$

where a_1 and a_2 are the model independent and model dependent axion degrees of freedom and F_1 and F_2 are the corresponding decay constants. Both F_1 and F_2 are related to the compactification scale and are about 10^{15} GeV [11]. F can be taken to be either the electromagnetic or weak or strong interactions field strength tensor. F' corresponds to hidden sector gauge field strength tensors. The lagrangian can be rewritten as

$$L = \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{2}(\partial_\mu a')^2 + \frac{1}{32\pi^2 F_a} a (g^2 F \tilde{F}) + \left(\frac{1}{32\pi^2 F'_a} a' \right) \left(g'^2 F' \tilde{F}' + g^2 \frac{F_2^2 - F_1^2}{F_1^2 + F_2^2} F \tilde{F} \right) \quad (2a)$$

where

$$a = \frac{F_1 a_1 + F_2 a_2}{(F_1^2 + F_2^2)^{1/2}}, \quad (2b)$$

$$a' = \frac{F_2 a_1 - F_1 a_2}{(F_1^2 + F_2^2)^{1/2}}, \quad (2c)$$

$$F_a = \frac{1}{2}(F_1^2 + F_2^2)^{1/2}, \quad (2d)$$

and

$$F'_a = F_1 F_2 / (F_1^2 + F_2^2)^{1/2}. \quad (2e)$$

a is the QCD axion and a' is the E'_8 axion. a' is massless at high temperatures. But it acquires a potential and a mass m as the universe cools. The low temperature ($T < \Lambda$) potential for a' is calculated to be [6,12]

$$V = \frac{\pi}{4M_{Pl}^2} \Lambda^6 |1 - \exp(ia'/15F'_a)|^2. \quad (3)$$

Therefore the low temperature mass of the axion a' is

$$m_0^2 = \frac{\pi}{450} \frac{\Lambda^6}{M_{Pl}^2 F_a'^2}. \quad (4)$$

a' decays to two gluons, photons or Z 's or to a W^\pm pair through the coupling $a' F \tilde{F}$. The gluons, W^\pm and Z particles predominantly create jets of mesons and some baryons. At $T \sim \Lambda$, the non-abelian subgroup in the hidden sector gets strong leading to the hadronization of particles. The hidden sector hadrons will have masses of the order of Λ . Therefore we can ignore decays to hidden sector particles. A large fraction of the energy released as mesons is transferred to the photons, either through scattering or annihilations. We shall assume that only 20 per cent of the energy of the decaying axions is transferred to the photons i.e. $f = 0.2$. This is an extremely conservative estimate.

The lifetime of the axion is given by

$$\tau = 8200 \pi^5 (g_1^4 + g_2^4 + g_3^4)^{-1} \frac{F_a'^2}{m_0^3} = 5.2 \times 10^{-17} \text{s} (g_1^4 + g_2^4 + g_3^4)^{-1} \pi^{7/2} \frac{F_a'^5 M_{Pl}^3 \text{GeV}}{\Lambda^9} \quad (5)$$

where $g_{1,2,3}$ are the couplings of $U(1)_Y$, $SU(2)_L$ and $SU(3)$ respectively. We have set $\frac{F_2^2 - F_1^2}{F_1^2 + F_2^2}$ equal to 1. We have checked that induced decays to photons and gluons due to coherent axion field oscillations [13,14,15,16] are inhibited due to Hubble expansion. For τ between 1 s and about 10^{13} s, the axion mass ranges between 10^4 GeV and about 10^{-1} GeV and the axion decays predominantly to mesons through gluons. For this range of lifetimes one can ignore g_1 and g_2 and let $g_3 = 1$ (or $\alpha_s \sim 0.1$). For lifetimes greater than about 10^{13} s the axion mass is less than twice the pion mass and the axion can only decay to photons. If one rewrites eqn. (2a) in terms of $F_{EM} \tilde{F}_{EM}$, one gets the lifetime by replacing $(g_1^4 + g_2^4 + g_3^4)^{-1}$ in eqn. (5) by $(4e^4)^{-1}$.

III

When the Peccei-Quinn symmetry breaks at early times the axion field need not be at the minimum of its potential. At a temperature T_i the field will start to oscillate about the minimum of its potential with a period $m^{-1}(T)$. (Above T_i the period of the oscillations is greater than the age of the universe H^{-1} . At high temperatures the mass has a temperature dependence, just as for the QCD axion.) We study the zero momentum mode of the E'_8 axion field which can be treated as a condensate of zero momentum particles. (Higher momentum modes are redshifted.) The energy density of the axion field today is given by [13,14,17]

$$\rho_{a_0} = m_0 n_0 = \frac{1}{2} m_0^2 A_0^2 \quad (6)$$

n_0 is the number density today and A_0 is the amplitude of the oscillations today. A classical treatment is permitted as the number density is large [18]. The energy density can be rewritten in terms of parameters at T_i when the field starts oscillating as

$$\rho_{a_0} = m_0 n_i \left(\frac{g_{s_0}}{g_{s_i}} \right) \frac{T_0^3}{T_i^3} = \frac{1}{2} m_0 m_i A_i^2 \left(\frac{g_{s_0}}{g_{s_i}} \right) \frac{T_0^3}{T_i^3}. \quad (7)$$

g_s is the effective number of relativistic degrees of freedom used to calculate the entropy. The above expression assumes that the axion field loses energy only due to the expansion of the universe and the variation of the mass with temperature and not due to decays. (The astrophysical constraints on massive decaying particles is given in terms of limits on the ratio of the energy density of the massive particles today, if they had not decayed, to the photon number density today.) T_i is determined by the condition $m_i = 3H$ [13].

$$T_i = \left[\frac{m_i M_{Pl}}{5g_{*i}^{1/2}} \right]^{1/2} \quad (8)$$

g_* is the effective number of relativistic degrees of freedom used to calculate the energy density. We shall take $g_{s_i} = g_{*i}$ and $g_{sf} = 3.9$. By the time the axion field starts oscillating, the axion mass has already attained its low temperature value in (4). One can verify this by checking that $m < 3H$ at $T = \Lambda$ when the field attains its low temperature mass (Appendix B). In deriving (8) we have assumed that the universe is radiation dominated when the axion field starts to oscillate. One can verify this by checking that $m > 3H$ at T_{eq} when $\rho_a = \rho_r$, i.e., the axion field starts oscillating before the axion energy density dominates the universe (Appendix C). Combining (7) and (8) gives

$$\rho_{a_0} = \frac{5^{3/2}}{2} \frac{g_{s_0}}{g_{*i}^{1/4}} m_0^{1/2} A_i^2 \frac{T_0^3}{M_{Pl}^{3/2}}. \quad (9)$$

A_i is taken to be $F'_a \simeq 10^{15}$ GeV. In a non-inflationary universe, the spatial average of A_i/F'_a is $\pi/\sqrt{3}$. In an inflationary universe, setting A_i equal to F'_a assumes that the universe lies in a typical post-inflationary region with $A_i/F'_a \sim 1$. Linde has criticized this assumption. We refer the reader to Ref. [19] for a discussion of this issue. Since the number density of photons $n_\gamma = (1.2/\pi^2)g_\gamma T^3$

$$\rho_a/n_\gamma \Big|_0 = 0.62 \pi^{9/4} \frac{\Lambda^{3/2} F_a'^{3/2}}{M_{Pl}^2}. \quad (10)$$

This is the ratio of the axion energy density today in the absence of decays to the photon number density today.

Our method will be to look at the astrophysical and cosmological constraints that apply to $\rho_a/n_\gamma|_0$ for different lifetimes τ . For lifetimes between 1 s and 1.1×10^7 s we calculate the range of Λ from (5) and substitute that in (10) and compare with the limits on the energy density from observations. We show that axions with lifetimes in this range are ruled out by cosmological arguments for $F'_a = 10^{15}$ GeV. Larger lifetimes are ruled out by limits on the distortions of the microwave and gamma ray backgrounds and by closure arguments. They would also give too small masses for the gauginos and supersymmetric scalars. τ less than 1 s corresponds to Λ greater than 1.2×10^{13} GeV.

IV

The presence of the axion has different effects during different epochs in the evolution of the universe. The presence of axions during the neutron-proton freeze-out (~ 1 s) affects the ratio of neutrons to protons at freeze-out and thereby leads to a change in the amount of helium in the universe. Their presence during light element synthesis (~ 1 minute) affects the amount of deuterium and ^3He . The decay of axions to photons or to other particles that transfer their energy to photons increases n_γ and dilutes η , the ratio of the baryon number density to the photon number density. This then alters the calculations of light element abundances. For $10^4 \text{ s} < \tau < 10^7 \text{ s}$, the decay photons interact with the background thermal photons and create a cascade of photons with enough energy to photodissociate deuterium and helium. For lifetimes between 10^5 s and (re)combination the decay photons change the shape of the spectrum of the cosmic background radiation. Decays occurring after recombination change the diffuse extra-galactic gamma ray spectrum. Decays of lifetimes greater than the age of the universe can lead to the axion energy density overclosing the universe.

We now explore the astrophysical constraints on the energy density of the axions for different lifetimes of the axion.

1 s < τ < 10^4 s: Increased expansion rate and dilution of η

The interaction rate for the weak reactions $n + \nu_e \leftrightarrow p + e^-$ and $p + \bar{\nu}_e \leftrightarrow n + e^+$ goes as $G_F^2 T^5$ while the expansion rate of the universe is $H = [(8\pi/3)\rho/M_{\text{Pl}}^2]^{(1/2)}$, where ρ is the energy density of the universe [20]. When the weak reactions are faster than the expansion of the universe the ratio of the neutrons to protons is given by

$$\frac{n}{p} = \exp\left[-\frac{\Delta m}{T}\right]. \quad (11)$$

Δm is the difference between the neutron and the proton mass. In the standard cosmology, at $t \sim 1 \text{ s}$ ($T \sim 1 \text{ MeV}$), the reaction rates become less than the expansion rate of the universe and the neutrinos freeze out and the protons and neutrons are no longer in

chemical equilibrium. Thereafter the neutron-proton ratio does not change from its equilibrium value at the freeze-out temperature, except for free neutron decays. The precise temperature at which the freeze-out occurs is determined by the expansion rate of the universe which is influenced by the energy density of the axion. At slightly later times between 1 and 3 minutes ($0.3 \text{ MeV} > T > 0.1 \text{ MeV}$), the light elements deuterium, ^3He , ^4He and ^7Li are produced.

An increase in the expansion rate due to the axions causes the weak reactions above to freeze out at an earlier temperature. This leads to a higher neutron to proton ratio and a higher percentage of ^4He , if all the neutrons are converted into helium. However, the deuterium and ^3He abundances are sensitive to the expansion rate at temperatures between 0.08 MeV and 0.04 MeV when the strong reactions that convert deuterium and ^3He into ^4He freeze out. The amount of deuterium, ^3He and ^4He produced also depends on η_{ms} , the baryon-to-photon ratio during nucleosynthesis [21]. For a given η_0 today, the baryon-to-photon ratio before the axions decay is higher for higher axion energy density. The amount of ^4He today increases for higher η_{ms} while the amount of deuterium and ^3He decreases because the rates of the reactions that convert deuterium and ^3He into ^4He are proportional to η_{ms} .

The constraint that ^4He makes up less than 25 percent of the energy density of the baryons in the universe and that the ratio of the abundances of deuterium and ^3He to the baryonic abundance is less than 1.0×10^{-4} gives us limits on $\rho_a/n_\gamma|_0$. For $1 \text{ s} < \tau < 10^4 \text{ s}$, the limit from light element abundances implies that $\rho_a/n_\gamma|_0$ is less than $1.5 \times 10^3 f^{-1} \text{ GeV}$ to $8.4 \times 10^{-5} f^{-1} \text{ GeV}$ [21]. But the axion energy density lies between $7.5 \times 10^4 \text{ GeV}$ and $1.6 \times 10^4 \text{ GeV}$. Therefore this range of lifetimes corresponding to Λ between $1.2 \times 10^{13} \text{ GeV}$ and $4.3 \times 10^{12} \text{ GeV}$ and the axion mass between $1.2 \times 10^4 \text{ GeV}$ and $5.5 \times 10^2 \text{ GeV}$ is ruled out.

$10^4 \text{ s} < \tau < 1.1 \times 10^7 \text{ s}$: Photodissociation

For decays that occur in the time interval $10^4 \text{ s} < \tau < 10^7 \text{ s}$, the decay photons can initiate electromagnetic cascades that can photodissociate deuterium and helium [22]. The high energy decay photons interact with the thermal photons producing electron-positron pairs. The electrons and positrons then lose their energy by inverse-Compton scattering the thermal photons to higher energies. These photons will repeat the above process till their energies fall below $E_* \simeq m_e^2/22T$, the threshold for pair production. (m_e is the electron mass.) Photons with energy greater than E_* and Q , the threshold for photodissociation of an element, are more likely to pair produce than interact with the nuclei of the light elements [22]. The spectrum of the electromagnetic cascade initiated by a decay photon

of energy $E(> E^*)$ is [23]

$$\frac{dN^E}{dE_\gamma} = \begin{cases} \frac{24}{55}\sqrt{2}\frac{E}{\sqrt{E_*}}E_\gamma^{-3/2} & \text{for } 0 \leq E_\gamma \leq E_*/2 \\ \frac{3}{55}EE_*^3E_\gamma^{-5} & \text{for } E_*/2 \leq E_\gamma \leq E_* \\ 0 & \text{for } E_* < E_\gamma, \end{cases} \quad (12)$$

where the cascade spectrum has been normalized as

$$\int_0^{E_*} E_\gamma \left(\frac{dN^E}{dE_\gamma} \right) dE_\gamma = E. \quad (13)$$

The photodissociation of an element starts when E_* becomes larger than the photodissociation threshold for that element.

The mass fraction of an element is defined as $X_k \equiv n_k Z_k / n_N$, where Z_k is the atomic number of the element, n_k is the number density of the element and n_N is the number density of the nucleons. The change in the mass fraction is given by [22,23]

$$\begin{aligned} \frac{dX_k}{dt} = -\frac{dn_a}{dt} \int_0^\infty \frac{dN}{dE} dE & \left[\int_{Q_k}^{E_*(t)} \frac{dN^E}{dE_\gamma} \frac{X_k \sigma_k}{n_e \sigma_C} dE_\gamma \right. \\ & \left. - \sum_{j \neq k} \int_{Q_j}^{E_*(t)} \frac{dN^E}{dE_\gamma} \frac{X_j \sigma_{j \rightarrow k}}{n_e \sigma_C} dE_\gamma \right] \end{aligned} \quad (14)$$

where $\sigma_{j \rightarrow k}$ is the partial cross-section for the dissociation of element j to k , σ_k is the total cross-section for photodissociation of element k and σ_C is the Compton scattering cross-section on the thermal electrons with density n_e . Anticipating the stringent conditions on the energy density of axions we assume above that n_e is not changed significantly by the decays and cascade process. $\frac{dn_a}{dt}$ includes the change in the number density of axion due to decays only (and not due to the expansion of the universe).

$$\frac{dn_a}{dt} = -\frac{n_a^I(t)}{\tau} \exp\left(-\frac{t}{\tau}\right) \quad (15)$$

where $n_a^I(t)$ is the value of n_a at some very early time t_I much less than τ , scaled to the time of dissociation t . $e^{-t_I/\tau} \sim 1$. $\frac{dN}{dE}$ is the spectrum of decay photons. To simplify our calculation we only include photons coming directly from the decay of the axions. $\frac{dN}{dE}$ is then $2\delta(E - m_0/2)$. The ratio of the decay rates into photons and into gluons is $\alpha_{EM}^2/\alpha_s^2 \sim 10^{-5}$. Therefore we include a factor of $f_\gamma = 10^{-5}$ below. Including all the energy transferred to photons via all the axion decay channels can only tighten our constraints.

Integrating (14) gives

$$X_k^f - X_k^i \simeq f_\gamma \frac{\rho_a}{n_\gamma} \bigg|_0 \frac{8}{7\eta_0} \left[-X_k^i \beta_k(\tau) + \sum_{j \neq k} X_j^i \beta_{j \rightarrow k}(\tau) \right] \quad (16a)$$

where

$$\beta_k(\tau) = \frac{1}{\tau} \int_{t_k^i}^{\infty} dt \exp\left(-\frac{t}{\tau}\right) \int_{Q_k}^{E_*(t)} \frac{1}{E} \frac{dN^E}{dE_\gamma} \frac{\sigma_k(E_\gamma)}{\sigma_C(E_\gamma)} dE_\gamma \quad (16b)$$

and

$$\beta_{j \rightarrow k}(\tau) = \frac{1}{\tau} \int_{t_j^i}^{\infty} dt \exp\left(-\frac{t}{\tau}\right) \int_{Q_j}^{E_*(t)} \frac{1}{E} \frac{dN^E}{dE_\gamma} \frac{\sigma_{j \rightarrow k}(E_\gamma)}{\sigma_C(E_\gamma)} dE_\gamma. \quad (16c)$$

i above represents the time when the photodissociation starts, i.e., when the cascade cutoff energy E_* equals the photodissociation threshold Q . η_0 is the baryon-to-photon ratio today. Above we have used

$$n_a^I(t)/n_e(t) = n_a^I(t_0)/n_e(t_0) \quad (17)$$

and

$$n_e(t_0) \simeq \frac{7}{8} n_{N_0} = \frac{7}{8} \eta_0 n_{\gamma_0}. \quad (18)$$

The subscript 0 refers to quantities today and N to nucleons.

t^i is calculated from

$$t^i = 1.6 \times 10^{-25} g_s^{-1/2} \frac{M_{Pl}^{7/4} \text{ GeV}}{m_0^{1/4} F_a'(m_e^2/22Q)^{3/2}} \quad (19)$$

where Q is 2.2 MeV for deuterium, 6.5 MeV for ^3He and 20 MeV for ^4He [24]. We have used the time-temperature relationship obtained in eqn.(11) of ref. [8] to obtain (19). g_s is 3.9. The photodissociation of light elements only goes on till the energy of the decay photons itself falls below the threshold for pair production. Then no cascade appears and photodissociation effects can be ignored. The time at which this occurs is calculated from

$$E = \frac{m_0}{2} = \frac{m_e^2}{22T} \quad (20a)$$

where

$$m_0 = 1.2 \times 10^{-6} \left[\frac{F_a'^2 \text{ GeV}}{(\tau/\text{s})} \right]^{1/3} \quad (20b)$$

(20b) is calculated from (5). Combining (20) with the $t - T$ relationship

$$T = \left[\frac{\text{s}}{t} \frac{1.6 \times 10^{-25} M_{Pl}^{7/4} \text{ GeV}}{g_s^{1/2}} \frac{1}{m_0^{1/4} F_a'} \right]^{2/3} \quad (21)$$

gives $t = 1.1 \times 10^7$ s. For decays that occur after this time, photodissociation of light elements is negligible.

For 10^4 s $< \tau < 1.1 \times 10^7$ s, the limits on $\rho_a/n_\gamma|_0$ come from the constraints on the increase in deuterium and ^3He due to photodissociation of ^4He . (The depletion of deuterium and ^3He is eclipsed by the increase in deuterium and ^3He due to photodissociation of ^4He as ^4He is 10^4 times more abundant.)

$$X^f(D + ^3\text{He}) - X^i(D + ^3\text{He}) \simeq \frac{8}{7} Y^i(^4\text{He}) \frac{\beta_{^4\text{He}}(\tau)}{\eta_0} r \frac{\rho_a}{n_\gamma} \Big|_0 f_\gamma, \quad (22)$$

where $r = (\frac{3}{4}\sigma_{\gamma^4\text{He} \rightarrow n^3\text{He}, p^3\text{H}} + \frac{1}{2}\sigma_{\gamma^4\text{He} \rightarrow npD})/\sigma_{\gamma^4\text{He} \rightarrow \text{all}} \simeq 0.5$.

β are computed numerically in Ref. [23] for a radiation dominated universe. The t^i in our matter dominated universe are much earlier than those calculated in Ref. [23]. Hence $\beta_{^4\text{He}}$ will be greater in our case for lifetimes between 10^4 s and 10^7 s. We take $\beta_{^4\text{He}}$ to be the peak value obtained in Ref. [23] of 1 GeV^{-1} . We take $Y^i(^4\text{He}) < .25$ which implies $\eta < 1.3 \times 10^{-9}$ and consequently $X^i(D + ^3\text{He}) > 3.4 \times 10^{-5}$. To be consistent with galactic chemical evolution $(D + ^3\text{He})/\text{H} < 10^{-4}$ or $X^f(D + ^3\text{He}) < 2.3 \times 10^{-4}$. Thus the upper limit on $\rho_a/n_\gamma|_0$ is $1.0 \times 10^{-12} f_\gamma^{-1} \text{ GeV}$. But the calculated value of $\rho_a/n_\gamma|_0$ is between $1.6 \times 10^4 \text{ GeV}$ and $5.1 \times 10^3 \text{ GeV}$. Hence the corresponding values of Λ between $4.3 \times 10^{12} \text{ GeV}$ and $2.0 \times 10^{12} \text{ GeV}$ are ruled out. The corresponding values of the axion mass lie between $5.5 \times 10^2 \text{ GeV}$ and 56 GeV .

$\tau > 1.1 \times 10^7$ s:

For lifetimes greater than 1.1×10^7 s, the decay photons would distort the cosmic microwave background or show up in the gamma ray background spectrum. Such lifetimes would also give supersymmetric masses less than 100 MeV . Lifetimes greater than the age of the universe would overclose the universe. Furthermore, if one assumes that the maximum baryon asymmetry $\eta \equiv n_B/n_\gamma$ produced in the early universe is $\text{O}(1)$ [25,26] and that the baryon asymmetry today is 3×10^{-10} , then constraints on the dilution of the universe imply that Λ must be greater than $7 \times 10^{12} \text{ GeV}$ [8].

Conclusion:

Thus we see that astrophysical and cosmological arguments imply that Λ is greater than $1.2 \times 10^{13} \text{ GeV}$ or that M_S is greater than 10^{10} GeV . Our result agrees with the value of $5 \times 10^{13} \text{ GeV}$ for Λ obtained independently by setting supersymmetric scalar masses equal to m_W .

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Appendix A

In this Appendix, we show that the universe is radiation dominated at the temperature $T \sim \Lambda$, when the axion attains its low temperature mass.

At $T = \Lambda$, the radiation energy density is given by

$$\rho_{rad} = \frac{\pi^2}{30} g_* \Lambda^4 \quad (A.1)$$

while the axion energy density is given by

$$\begin{aligned} \rho_a &= m_0 n = \frac{1}{2} m_0^2 A^2(\Lambda) \\ &= \frac{\pi}{900} \Lambda^4 \frac{\Lambda^2}{M_{Pl}^2} \frac{A^2(\Lambda)}{F_a'^2} \end{aligned} \quad (A.2)$$

The low temperature mass m_0 used above is given by (4). g_* is 106.75. At $T = \Lambda$ the field has not started oscillating (see Appendix B) and we assume that $A(\Lambda)$ is approximately F_a' . Also, $\Lambda < M_{Pl}$. Thus, $\rho_a < \rho_{rad}$ at $T = \Lambda$.

Appendix B

In this Appendix we show that the axion field starts oscillating at a temperature T_i below $T = \Lambda$ at which the axion attains its low temperature mass. We shall show below that $m < 3H$ at $T = \Lambda$, i.e., the axion field has not yet started oscillating at $T = \Lambda$.

Since the universe is still radiation dominated at $T = \Lambda$ (see Appendix A),

$$3H = 0.9\pi^{3/2} g_*^{1/2} \frac{\Lambda^2}{M_{Pl}} \quad (B.1)$$

However,

$$m_0 = \left(\frac{\pi}{450} \right)^{1/2} \frac{\Lambda^2}{M_{Pl}} \frac{\Lambda}{F_a'} \quad (B.2)$$

Since $\Lambda < F_a'$ and $g_* = 106.75$, $m < 3H$ at $T = \Lambda$. Thus the axion field starts oscillating at a temperature $T_i < \Lambda$, when H has decreased sufficiently to satisfy $m = 3H$.

Appendix C

In this Appendix we show that the universe is still radiation dominated at the temperature T_i when the axion field starts oscillating. We actually show that the axion field is already oscillating by the time the temperature has decreased to T_{eq} , when the axion energy density is equal to the radiation energy density.

At $T = T_{\text{eq}}$,

$$3H = (12\pi)^{1/2} \frac{A(T_{\text{eq}})}{M_{Pl}} m_0 \quad (C.1)$$

Since $A(T_{\text{eq}}) \leq A(T_i) = F'_a < 0.2M_{Pl}$, $m > 3H$ at T_{eq} . This implies that the universe started oscillating at a temperature greater than T_{eq} , when the universe was still radiation dominated.

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